How Much Do You Weigh? (Hint: It's a Trick Question)

G. Bélanger, PhD, European Space Astronomy Centre, Spain

A friend of mine called this morning to ask a favour. Someone he knows read something about the upcoming eclipse on a NASA webpage, and got the impression that there was a mistake in their calculations. The passage was the following:

> *Starting as an observer on the ground, you are under the gravitational influence of Earth, the moon and the sun. At the time of the August 21, 2017 eclipse, Earth will be 151.4 million kilometers from the sun, and the moon will be located 365,649 km from the surface of Earth. Using Newton's Law of Gravity, we can calculate the force of the sun, moon and Earth on an 80 kg person. Earth accounts for 784.1 Newtons of force (176.42 pounds), the moon provides 0.0029 Newtons (0.01 ounces) and the sun provides 0.4633 Newtons (1.6 ounces). But because our Earth rotates, this also provides an 'anti-gravity' centrifugal force we can also calculate. So if we add the forces with their correct directions we get a total gravitational force of 784.1 – 0.0029 – 0.4633 = 783.634 Newtons or 176.317 pounds. So, you will be about 1.7 ounces lighter!*

My friend asked me to check it out and report back to him so that he could pass on the information. So I did, and I thought it might be interesting to write it up, just in case you may be interested, and in case it happens again at some point in the future.

Consider an object of mass m sitting on the surface of the Earth. The relevant forces acting on it are the gravitational forces due to 1) the **Earth**, 2) the **Moon**, and 3) the **Sun**. Other forces either due to other planets or to the fictitious centrifugal force are negligible compared to these three. Newton's equation for the gravitational force is

$$F_g = \frac{GMm}{r^2}, \tag{1}$$

where G is the gravitational constant, M and m are the two masses invovled (M is usually used for the larger one, and m the smaller), and r is the distance between them.

We have three bodies for which we need masses and radii/distances (SI). The masses are:

$$\begin{aligned} M_\oplus &= 5.972 \times 10^{24} \text{ kg} \\ M_☾ &= 7.348 \times 10^{22} \text{ kg} = \mathbf{0.0123\ M_\oplus} \\ M_\odot &= 1.989 \times 10^{30} \text{ kg} = \mathbf{332946\ M_\oplus} \end{aligned}$$

And the radii (R) or distances (d) are:

$$\begin{aligned} R_\oplus &= 6.371 \times 10^{6} \text{ m} \\ d_☾ &= 384400 \text{ km} = \mathbf{60.336\ R_\oplus} \\ d_\odot &= 1.496 \times 10^{11} \text{ m} = \mathbf{23481\ R_\oplus} \end{aligned}$$

Writing down the equation for the sum of the forces, note that the Earth exerts an attractive force towards its centre, whereas the Moon and the Sun are pulling away from the Earth towards them. Therefore, let's take the Earth's force to be positive (pulling towards the surface) and the other two as negative (pulling away from the surface).

$$F = F_\oplus - F_☾ - F_\odot \tag{2}$$

Expanding this we get:

$$\begin{aligned} F &= \frac{GM_\oplus m}{R_\oplus^2} - \frac{GM_☾ m}{d_☾^2} - \frac{GM_\odot m}{d_\odot^2} \\ &= Gm\left(\frac{M_\oplus}{R_\oplus^2} - \frac{M_☾}{d_☾^2} - \frac{M_\odot}{d_\odot^2}\right) \\ &= Gm\left(\frac{M_\oplus}{R_\oplus^2} - \frac{0.0123M_\oplus}{(60.336R_\oplus)^2} - \frac{332946M_\oplus}{(23481R_\oplus)^2}\right) \end{aligned} \tag{3}$$

and finally we end up with this

$$F = F_\oplus\left(1 - 3.379\times10^{-6} - 6.039\times10^{-4}\right) \tag{4}$$

which tells us that, with respect to the gravitational force exerted by the Earth ($F_\oplus$), the Sun's gravitational pull on the object at the Earth's surface is barely 6 ten thousandths, and the Moon's is just 3.3 millionths of the Earth's force. Therefore, the gravitational force that the Sun exerts is about *200 times greater than the Moon's*, but both are very, very small compared to the Earth's:

$$\begin{aligned} \text{Sun: } F_\odot &= 6.039 \times 10^{-4}\, F_\oplus \\ \text{Moon: } F_☾ &= 3.379 \times 10^{-6}\, F_\oplus \end{aligned}$$

A person standing on Earth with a mass of 80 kg would weigh 785.6 N (newtons), because weight is defined as mass times the acceleration due to gravity at sea level, g_0, given by:

$$g_0 = \frac{GM_\oplus}{R_\oplus^2}\ \text{m}\,\text{s}^{-2} \qquad (5)$$

The most recent official value of the gravitational constant is (CODATA 2014):

$$G = 6.67408 \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2},$$

and therefore, the value of g_0 is

$$g_0 = \frac{(6.67408 \times 10^{-11})(5.972 \times 10^{24})}{(6.371 \times 10^6)^2} = 9.81965$$

Naturally, because this value depends on the distance to the centre of the Earth, it depends on our altitude with respect to sea level. At higher altitudes, the distance increases, the denominator grows larger, and the value of g decreases.

The top of Mount Everest, the highest point on Earth, is at 8850 m above sea level. There,

$$g_{\min} = \frac{(6.67408 \times 10^{-11})(5.972 \times 10^{24})}{(6.371 \times 10^6 + 8850)^2} = 9.79243.$$

And so instead of weighing 785.6, we would weigh 783.4 N, or 0.28% and 224 g less than at sea level. And using (4), we can say that the forces from the Sun and Moon on the 80 kg person would be:

$$\begin{aligned} F_\odot &= (6.039 \times 10^{-4})(785.6) = 0.474 \text{ N or } 48 \text{ g} \\ F_☾ &= (3.379 \times 10^{-6})(785.6) = 0.00265 \text{ N or } 0.3 \text{ g} \end{aligned}$$

If we now consider the effect variations in distance, because the orbits are not circular but elliptical, we find that for the Moon, these differences are small: the minimum distance from the Earth is 356500 km and the maximum is 406700 km. For the Sun, the minimum distance (perihelion) is at 147.1, and the maximum (aphelion) at 152.1 million km. These translate into changes of the gravitational pull that they have on an object at the surface of the Earth. Let's see by how much.

The Sun, has about 200 times the Moon's gravitational pull. We used 149.6×10^9 m for the Earth-Sun distance, which we saw was 23481 $R_\oplus$. At its most distant, the Sun is at 23874 $R_\oplus$, and at its closest it is at 23089 $R_\oplus$. Using once more (3), we find:

$$\begin{aligned} F_\odot^{\min} &= \frac{332946}{(23874)^2}\, F_\oplus, & (6) \\ &= 5.842 \times 10^{-4}\, F_\oplus, \ \text{ at } 152.1 \times 10^9\text{m}, \\ F_\odot^{\max} &= \frac{332946}{(23089)^2}\, F_\oplus, & (7) \\ &= 6.246 \times 10^{-4}\, F_\oplus, \ \text{ at } 147.1 \times 10^9\text{m}. \end{aligned}$$

These variations amount to:

$$\begin{aligned} F_\odot^{\min} &= (5.842 \times 10^{-4})(785.6) = 0.459 \text{ N or } 47 \text{ g} \\ F_\odot^{\max} &= (6.246 \times 10^{-4})(785.6) = 0.490 \text{ N or } 50 \text{ g} \end{aligned}$$

The Moon, as we saw, has a very small gravitational pull compared to the Sun's, which, at its average distance of of 60.336 $R_\oplus$, corresponds to 3.379×10^{-6} $F_\oplus$. At its furthest, it is at a distance of 63.836 $R_\oplus$, and at its closest, it is at 55.957 $R_\oplus$. This means that:

$$\begin{aligned} F_☾^{\min} &= \frac{0.0123}{(63.836)^2}\, F_\oplus, & (8) \\ &= 3.018 \times 10^{-6}\, F_\oplus, \ \text{ at } 4.067 \times 10^8\text{m}, \\ F_☾^{\max} &= \frac{0.0123}{(55.957)^2}\, F_\oplus, & (9) \\ &= 3.928 \times 10^{-6}\, F_\oplus, \ \text{ at } 3.565 \times 10^8\text{m}. \end{aligned}$$

In this case, these variations amount to:

$$\begin{aligned} F_☾^{\min} &= (3.018 \times 10^{-6})(785.6) = 0.00237 \text{ N or } 0.24 \text{ g} \\ F_☾^{\max} &= (3.928 \times 10^{-6})(785.6) = 0.00309 \text{ N or } 0.31 \text{ g} \end{aligned}$$

Therefore, in conclusion, we can say at least the following in regards to the gravitational influences of the Sun and Moon on the Earth. Roughly speaking, the Moon has a mass of 0.12 $M_\oplus$, and orbits at 60 $R_\oplus$; the Sun has a mass of 333 k$M_\oplus$, and is 23.5 k$R_\oplus$ away; and this makes the Sun's influence 200 times greater than the Moon's. Weight on Earth is measured in newtons, and defined based on to the value of gravitational acceleration at sea level g_0 (5). A person of 80 kg at the top of the Everest would weigh 224 g less than at sea level; the Sun's pull on them is on average equivalent to 48 g, ranging between 47 and 50 g depending on the Sun's distance; and the Moon's pull on them is on average of 0.3 g, ranging from 0.24 to 0.31 g depending on the Moon's distance.

And so now, can you tell me if there was or not a mistake in that paragraph on the NASA webpage? And can you tell me precisely much to you weigh?